Quiz 2

Be sure to show work clearly.

(1) Given the points P(9, 3 ,-1) and Q(1,4,7)　　　(4 points)

　　a). Find the distance between P and Q.　　$\sqrt{114}$

　　b) Find the vector $\overrightarrow{PQ}$　　$\langle -7, 1, 8 \rangle$

(2) Given the vectors $\mathbf{a} = \langle -5,1,2 \rangle$ and $\mathbf{b} = \langle -1,0,4 \rangle$, find

　　a) the angle between $\mathbf{a}$ and $\mathbf{b}$　　(2 points)
　　(Note: In this class, exact answers should always be given unless otherwise stated; that is

　　don't use your calculator to get an approximation)

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\,\|\vec{b}\|} = \frac{13}{\sqrt{30}\sqrt{17}}$$
$$\theta = \cos^{-1}\left(\frac{13}{\sqrt{510}}\right)$$

　　b) $\mathbf{a} \times \mathbf{b}$　　(10 points)

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & 1 & 2 \\ -1 & 0 & 4 \end{vmatrix} = \langle 4, 18, 1 \rangle$$

Check your answer by showing it is orthogonal to both a and b

$$\langle 4, 18, 1 \rangle \cdot \langle -5, 1, 2 \rangle = 0$$
$$\langle 4, 18, 1 \rangle \cdot \langle -1, 0, 4 \rangle = 0$$

Note: $\vec{a} \times \vec{b}$ is always orthogonal to both $\vec{a}$ and b, so these should be zero. I suggest doing this on all your cross products

　　c) $\text{proj}_{\mathbf{a}}\mathbf{b}$　　(2 points)

$$\text{proj}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\,\vec{a}$$
$$\left\langle \frac{-13}{6}, \frac{13}{30}, \frac{13}{15} \right\rangle$$

Projection on $\vec{a}$ is a multiple of $\vec{a}$

　　d) a unit vector in the direction of $\mathbf{b}$　　(2 points)

$$\vec{u} = \frac{1}{\|\vec{b}\|}\vec{b}$$
$$\left\langle -\frac{1}{\sqrt{17}}, 0, \frac{4}{\sqrt{17}} \right\rangle$$